Introduction

• Before getting started with electronic circuits this chapter will spend some time on math fundamentals, metric prefixes, relevant scientific units, and a few constants
• This will help set the stage for what lies ahead

Mathematics

In this section I will review some basic algebra and equation solving, working with exponents and logarithms, and examples in Excel.

Algebra and Equation Solving

• Simple little equations pop up frequently in electronic circuit design

• Equal ratios: Consider

\[
\frac{a}{b} = \frac{c}{d}
\]  

(2.1)

– You want to find a value for \(d\) in terms of the other three quantities so cross multiply and divide both sides by \(a\)
\[ ad = bc \Rightarrow d = \frac{bc}{a} \] \hspace{1cm} (2.2)

- **Linear equation:** Consider
  \[ 5x + 45 = 62 \] \hspace{1cm} (2.3)

Find \( x \) to satisfy the above equation
- The most direct approach is to subtract 45 from both sides and then divide both sides by 5:
  \[
  (5x + 45) - 45 = 62 - 45 \\
  5x = 17 \\
  x = \frac{17}{5} = 3.40
  \] \hspace{1cm} (2.4)

- **Quadratic equation:** Consider
  \[ ax^2 + bx + c = 0 \] \hspace{1cm} (2.5)

  - There are two solutions to this equation
  - You can use the quadratic formula to solve for \( x \)
    \[
    x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
    \] \hspace{1cm} (2.6)

  - Unfortunately the quadratic may have two *real* number solutions or a pair *complex* number solutions, depending upon the *sign* of \( b^2 - 4ac \)
  - In this class you will only see real solutions
Exponents

• In dealing with algebra problems involving exponents (numbers or variables raised to a power) sometimes pop up, so it is important to remember some laws of exponents:
  
  – For any constant $a$,
    \[ a^1 = a \quad a^0 = 1 \]
    \[(2.7)\]

  – When adding, subtracting, multiplying and dividing numbers you in general evaluate the exponents first, then do the operation, i.e.,
    \[ a^x \cdot b^y = (a^x) \cdot (b^y) \]
    \[(2.8)\]

  – If the $a = b$, then you can manipulate the exponents when multiplication and division are involved, i.e.,
    \[ a^x \cdot a^y = a^{x+y} \]
    \[ \frac{a^x}{a^y} = a^{x-y} \]
    \[(2.9)\]

  – Other variations
    \[ (a^x)^y = a^{xy} \]
    \[(2.10)\]
    \[ (ab)^m = a^m b^m \]

Logarithms

• A logarithm is really just an exponent or power that a number
is raised to

- In electronic circuits powers of 10 are the most common, i.e.,
  \[ N = 10^x \]
  so \( x = \log_{10}(N) \) (2.11)

- As a specific example suppose \( N = 10000 \), then
  \[ x = \log_{10}(10000) = 4, \]
  since \( 10^4 = 10000 \) (2.12)

- The function \( \log_{10}(\ ) \) finds the base 10 logarithm

- The natural logarithm defines \( e = 2.71828 \) as its base
  - The natural log is given the special notation
    \[ \log_e(\ ) = \ln(\ ) \] (2.13)
  - If you have a calculator that does not have a \( \log_{10}(\ ) \) function you can always find the log of any base, including 10 using
    \[ \log_b(z) = \frac{\ln(z)}{\ln(b)} \] (2.14)

- The **Decibel** (named after Alexander Graham Bell) is used frequently in electronic circuit performance modeling

- The quantity of interest is the decibel (dB), which is defined as \( 10 \log_{10} \) the ratio of two power levels, i.e.,
dB = 10\log_{10}\left(\frac{P_2}{P_1}\right) \quad (2.15)

- If say $P_1$ is the input power to an amplifier and $P_2$ the output power, you can say that the amplifier power gain is

$$G_p = 10\log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \text{ dB} \quad (2.16)$$

- A generalization to power gain is the voltage gain, $G_v$, of a circuit (think back to the brief introduction to the common emitter amplifier), you can write that

$$G_v = 10\log_{10}\left(\frac{v_{\text{out}}^2}{v_{\text{in}}^2}\right) = 20\log_{10}\left(\frac{v_{\text{out}}}{v_{\text{in}}}\right) \quad (2.17)$$

- **Note**: If the resistance levels on the input and output side of the amplifier are equal, the voltage gain and power gain are equivalent

- **Special Cases of Interest:**
\[ 20 \log_{10} \left( \frac{2v_{in}}{v_{in}} \right) = 20 \log_{10}(2) = 6.026 \text{ dB} \]

\[ 20 \log_{10} \left( \frac{\sqrt{2}v_{in}}{v_{in}} \right) = 20 \log_{10}(\sqrt{2}) = 3.01 \text{ dB} \]

\[ 20 \log_{10} \left( \frac{v_{in}}{\sqrt{2}v_{in}} \right) = 20 \log_{10}(\frac{1}{\sqrt{2}}) = -3.01 \text{ dB} \]

\[ 20 \log_{10} \left( \frac{10v_{in}}{v_{in}} \right) = 20 \log_{10}(10) = 20 \text{ dB} \]
Spreadsheet (Excel) Examples

- Plot the parabola

\[ y = f(x) = -5x^2 + 25x + 10 = 0 \]  \hspace{1cm} (2.19)

- Plot the voltage gain function

\[ G(x) = \frac{1}{\sqrt{1 + (x/100)^2}} \]  \hspace{1cm} (2.20)

\[ G_{dB}(x) = 20\log_{10}[G(x)], \text{ for } 1 \leq x \leq 10^4 \]

with \( x \) a log base 10 axis

- The above examples can be found worked out in the spreadsheet chapter2.xlsx

- Captures of specific details used to plot (2.19) and (2.20) are shown below

**Parabola Plot:**

- A table is constructed using the cell function formula

\[ =-5*POWER(A2,2)+25*A2+10 \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x values</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>-5</td>
</tr>
<tr>
<td>4</td>
<td>-4.95</td>
</tr>
<tr>
<td>5</td>
<td>-4.9</td>
</tr>
<tr>
<td>6</td>
<td>-4.85</td>
</tr>
<tr>
<td>7</td>
<td>-4.8</td>
</tr>
<tr>
<td>8</td>
<td>-4.75</td>
</tr>
<tr>
<td>9</td>
<td>-4.7</td>
</tr>
</tbody>
</table>

- The base plot is a scatter plot with connected points
– When finally formatted the result is

![Plot of y = f(x) = -5x^2 + 25x + 10](image)

- **Gain Plot in dB:**
  – Again, a table is constructed as before, but to get log-spaced plot values an intermediate column is created.

<table>
<thead>
<tr>
<th>x values</th>
<th>y values</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00E+00</td>
</tr>
<tr>
<td>0.05</td>
<td>1.26E-04</td>
</tr>
<tr>
<td>0.1</td>
<td>1.41E-04</td>
</tr>
</tbody>
</table>

1. **Intermediate column:**

   \[ y_{\text{dB}} = 20 \times \log_{10} \left( \frac{y}{y_{\text{ref}}} \right) \]

2. **Final column:**

   \[ G(x) = 20 \times \log_{10} \left( \frac{1}{\sqrt{1 + \left( \frac{B2/100.2}{10} \right)^2}} \right) \]
- The completed plot is:
Metric Prefixes

- When using the metric system, units are always taken in multiples of 10 times larger or smaller, and so-on
- Metric prefixes are used in electronic circuit design and in describing component values
- Common metric prefixes are given in the table below:

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Multiplication Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>tera</td>
<td>T</td>
<td>$10^{12} = 1,000,000,000,000$</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>$10^9 = 1,000,000,000$</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>$10^6 = 1,000,000$</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>$10^3 = 1,000$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10^0 = 1$</td>
</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>$10^{-2} = 0.01$</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>$10^{-3} = 0.001$</td>
</tr>
<tr>
<td>micro</td>
<td>m</td>
<td>$10^{-6} = 0.000001$</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>$10^{-9} = 0.000000001$</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>$10^{-12} = 0.000000000001$</td>
</tr>
<tr>
<td>femto</td>
<td>f</td>
<td>$10^{-15} = 0.000000000000001$</td>
</tr>
</tbody>
</table>
Measurement Units

- Metric units are used internationally and constitute the *International System of Units* (SI)
- There are two categories of SI units: *Fundamental* and *Derived*
- Of the fundamental types four are the most relevant
  - **Distance**: unit name meter, symbol m
  - **Mass**: unit name kilogram, symbol kg
  - **Time**: unit name second, symbol s
  - **Electric current**: unit name ampere, symbol A
- There are many derived units relevant to electronic circuits, the major quantities and few minor quantities are given in the table below:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Name</th>
<th>Symbol</th>
<th>In other Terms</th>
<th>In fund. Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>hertz</td>
<td>Hz</td>
<td>cycles/s</td>
<td></td>
</tr>
<tr>
<td>Power</td>
<td>watt</td>
<td>W</td>
<td>$\frac{V^2}{\Omega}$, A V</td>
<td>$\frac{m^2 \text{kg}}{s^3}$</td>
</tr>
<tr>
<td>Electric charge</td>
<td>coulomb</td>
<td>C</td>
<td>s A</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 2 • Mathematics, Units, and Constants

Table 2.2: SI Derived Units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Name</th>
<th>Symbol</th>
<th>In other Terms</th>
<th>In fund. Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromotive force, voltage</td>
<td>volt</td>
<td>V</td>
<td>$A \Omega, \frac{W}{A}$</td>
<td>$\frac{m^2 kg}{s^3 A}$</td>
</tr>
<tr>
<td>Electric resistance</td>
<td>ohm</td>
<td>$\Omega$</td>
<td>$V/A$</td>
<td>$\frac{m^2 kg}{s^3 A^2}$</td>
</tr>
<tr>
<td>Capacitance</td>
<td>farad</td>
<td>F</td>
<td>$C/V$</td>
<td>$\frac{s^4 A^2}{m^2 kg}$</td>
</tr>
<tr>
<td>Inductance</td>
<td>henry</td>
<td>H</td>
<td>$\frac{V s}{A}$</td>
<td>$\frac{m^2 kg}{s^2 A^2}$</td>
</tr>
</tbody>
</table>

Constants

- A couple of constants used in communications circuit design include:
  - The speed of light in *free space*: $3 \times 10^8$ m/s
  - $\pi = 3.1415...$
  - The *natural number is* $e = 2.71828...$ and is the base used in the *natural logarithm* discussed earlier
References
